UDC 519.2

**DOI: 10.20998/2079-0023.2022.02.04**

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## ASSESSING THE INFORMATIVENESS OF THE CONTROLLED PARAMETERS IN THE TASK OF IDENTIFYING THE STATE OF THE SYSTEM

The effectiveness of solving the problem of identifying the system state significantly depends on the number of controlled parameters and the degree of their informativeness. The traditional method for assessing the informativeness of these parameters is based on the measure of distance between the probability distributions of the values of the controlled parameter for different states of the system proposed by Kullback. The shortcomings of Kullback measure have been revealed. Firstly, the value of this measure is not normalised and is not limited from above. Secondly, this measure is asymmetric, i.e. its numerical value depends on the way its components enter the calculation ratio. The method for calculating the informativeness criterion proposed in this paper takes into account the uncertainty that arises due to the fuzzy description of the boundaries of the areas of possible values of the controlled parameters for each of the possible states of the system. An important enhancement of the known methods for assessing the informativeness of the controlled parameters is to take into account the real existing inaccuracy in estimating the values of the results of measuring these parameters themselves. These circumstances determine the subject and purpose of the study that is the development of a method for calculating the distance between the distributions of fuzzy values of the controlled parameter, free from the shortcomings of the Kullback measure. To calculate the measure of the distance between the distributions of the values of the controlled parameter under conditions of uncertainty of the initial data, described in terms of fuzzy mathematics, a symmetric criterion is proposed, which is easily calculated. Examples of the criterion calculation are given. The possibilities of increasing the level of informativeness of the criterion using analytical descriptions of membership functions of fuzzy values of the controlled parameter for different states of the system are considered.

**Keywords:** identification of system states, Kullback information measure, assessment of informativeness of fuzzy controlled parameter.

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## ОЦІНКА ІНФОРМАТИВНОСТІ КОНТРОЛЬОВАНИХ ПАРАМЕТРІВ У ЗАВДАННІ ІДЕНТИФІКАЦІЇ СТАНУ СИСТЕМИ

Ефективність розв’язання задачі ідентифікації стану системи суттєво залежить від кількості контрольованих параметрів і ступеня їхньої інформативності. Традиційна технологія оцінювання інформативності цих параметрів спирається на запропоновану Кульбаком міру відстані між ймовірнісними розподілами значень контрольованого параметра для різних станів системи. Виявлені недоліки міри Кульбака. По-перше, значення цієї міри не нормоване і не обмежене зверху. По-друге, ця міра асиметрична, тобто її чисельне значення залежить від способу входження її компонентів у розрахункове співвідношення. Запропонована в роботі технологія розрахунку критерію інформативності враховує невизначеність, що виникає внаслідок нечіткості опису меж областей можливих значень контрольованих параметрів для кожного з можливих станів системи. Важливе посилення відомих технологій оцінювання інформативності контрольованих параметрів полягає у врахуванні реально існуючої неточності оцінювання значень самих результатів вимірювання цих параметрів. Зазначені обставини визначають предмет і мету дослідження – розроблення метода розрахунку відстані між розподілами нечітких значень контрольованого параметра, вільного від недоліків міри Кульбака. Для розрахунку міри відстані між розподілами значень контрольованого параметра в умовах невизначеності вихідних даних, описаних у термінах нечіткої математики, запропоновано симетричний критерій, який легко обчислюється. Наведено приклади розрахунку критерію. Розглянуто можливості збільшення рівня інформативності критерію з використанням аналітичних описів функцій належності нечітких значень контрольованого параметра для різних станів системи.

**Ключові слова:** ідентифікація станів системи, інформаційна міра Кульбака, оцінка інформативності нечіткого контрольованого параметра.

**Introduction.** Assessing the state of the system based on the results of processing a set of controlled parameters is a typical task of everyday practice. The elementary mathematical model of this problem is formulated as follows.

It is assumed that the system can be in one of the many $(H_1, H_2, \dots, H_m)$ states, for the identification of which the parameters $x = (x_1, x_2, \dots, x_n)$ are used.

Let from theoretical considerations (or based on the results of processing preliminary observations) a matrix of conditional distribution densities of random values of controlled parameters for possible states of the system be obtained: $f(x_j / H_i)$, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$.

It is clear that the task of analyzing the results of observations is the simpler, the smaller the number of controlled parameters.

The natural way to reduce this number is to estimate the informational value of the parameters and select the best ones.

**Analysis of known results.** One of the traditional approaches is the calculation and comparison of the “distance” between the distributions of random values of controlled parameters for various system states using the Kullback measure [1, 2].

This measure is introduced as follows. To assess the information value of a specific controlled parameter $x$ used to identify system states, for example, $H_1$ and $H_2$ the Kullback numerical criterion is calculated by the formula (1):

$$\tau_{1,2} = F(f(x / H_1), f(x / H_2)) = \\ = \int_{-\infty}^{\infty} f(x / H_1) \log \frac{f(x / H_1)}{f(x / H_2)} dx. \qquad (1)$$

In a particular case, for Gaussian distributions

$$f(x/H_1)=\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left\{-\frac{(x-m_1)^2}{2\sigma_1^2}\right\};$$

$$f(x/H_2)=\frac{1}{\sqrt{2\pi}\sigma_2}\exp\left\{-\frac{(x-m_2)^2}{2\sigma_2^2}\right\}.$$

We have

$$\tau_{1,2}=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left\{-\frac{(x-m_1)^2}{2\sigma_1^2}\right\}\times$$

$$\times\log\frac{\sigma_2}{\sigma_1}\frac{\exp\left\{-\frac{(x-m_1)^2}{2\sigma_1^2}\right\}}{\exp\left\{-\frac{(x-m_2)^2}{2\sigma_2^2}\right\}}dx=$$

$$=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left\{-\frac{(x-m_1)^2}{2\sigma_1^2}\right\}\log\frac{\sigma_2}{\sigma_1}\times$$

$$\times\exp\left\{-\left[\frac{(x-m_1)^2}{2\sigma_1^2}-\frac{(x-m_2)^2}{2\sigma_2^2}\right]\right\}dx=$$

$$=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left\{-\frac{(x-m_1)^2}{2\sigma_1^2}\right\}\left[\log\frac{\sigma_2}{\sigma_1}-\right.$$

$$\left.-\left[\frac{(x-m_1)^2}{2\sigma_1^2}-\frac{(x-m_2)^2}{2\sigma_2^2}\right]\right]dx=$$

$$=\log\frac{\sigma_2}{\sigma_1}-\frac{1}{2}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left\{-\frac{(x-m_1)^2}{2\sigma_1^2}\right\}L(x)dx,$$

$$L(x)=\frac{1}{\sigma_1^2\sigma_2^2}\left(x^2\sigma_2^2-2xm_1\sigma_2^2+m_1^2\sigma_2^2-\right.$$

$$\left.-x^2\sigma_1^2+2xm_2^2\sigma_1^2-m_2^2\sigma_1^2\right)=$$

$$=\frac{x^2(\sigma_2^2-\sigma_1^2)-2x(m_1\sigma_2^2-m_2\sigma_1^2)+m_1^2\sigma_2^2-m_2^2\sigma_1^2}{\sigma_1^2\sigma_2^2},$$

$$\tau_{1,2}=\log\frac{\sigma_2}{\sigma_1}-$$

$$-\frac{1}{2}\left[\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left\{-\frac{(x-m_1)^2}{2\sigma_2^2}\right\}\frac{(\sigma_2^2-\sigma_1^2)x^2}{\sigma_1^2\sigma_2^2}dx-\right.$$

$$-2\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left\{-\frac{(x-m_1)^2}{2\sigma_1^2}\right\}\frac{(m_1\sigma_2^2-m_2\sigma_1^2)x}{\sigma_1^2\sigma_2^2}dx+$$

$$\left.+\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left\{-\frac{(x-m_1)^2}{2\sigma_1^2}\right\}\frac{(m_1^2\sigma_2^2-m_2^2\sigma_1^2)}{\sigma_1^2\sigma_2^2}dx\right]=$$

$$=\log\frac{\sigma_2}{\sigma_1}-\frac{1}{2}\left(A_1-2A_2+A_3\right);$$

$$A_1=\frac{\sigma_2^2-\sigma_1^2}{\sigma_1^2\sigma_2^2}\left(m_1^2+\sigma_1^2\right);$$

$$A_2=\frac{m_1\sigma_2^2-m_2\sigma_1^2}{\sigma_1^2\sigma_2^2};$$

$$A_3=\frac{m_1^2\sigma_2^2-m_2^2\sigma_1^2}{\sigma_1^2\sigma_2^2}.$$

Then

$$\tau_{1,2}=\log\frac{\sigma_2}{\sigma_1}-\frac{1}{2\sigma_1^2\sigma_2^2}\left[\sigma_1^2\left(m_1^2-m_2^2\right)-\right.$$
$$\left.-\sigma_1^2\left(\sigma_2^2-\sigma_1^2\right)-2\left(m_1^2\sigma_2^2-m_2^2\sigma_1^2\right)\right]. \quad (2)$$

The approach proposed by Kullback to assessing the informativeness of indicators has obvious shortcomings. The most significant of them is the asymmetry of the introduced ratio, which leads to unpredictable differences in the results of calculating the measure $\tau$ when the nature of the occurrence of $f(x/H_1)$ and $f(x/H_2)$ in formula (1) changes. Secondly, the measure of informativeness of (1) is not normalized. Numerical value $\tau$ is equal to zero, if the distribution densities $f(x/H_1)$ and $f(x/H_2)$ coincide, and can take on an arbitrarily large, not limited from above, value otherwise [3–5]. Thirdly, the analytical complexity of the construction of criterion (1) leads in many practical cases to the need to use numerical integration [6, 7]. Finally, fourthly, the Kullback measure is designed to distinguish between distributions of random variables and cannot be used directly if the uncertainty of the initial data is described differently, for example, in terms of fuzzy set theory. This situation requires special consideration. The point is the following. The functioning of real systems, as a rule, occurs in changing conditions. At the same time, the mechanism of formation of the observed values of the controlled parameters of the environment and the system changes. As a result, the axiomatic requirements of probability theory are violated. Under these conditions, the level of adequacy of empirical distribution densities obtained by continuous approximation of histograms formed from the results of experiments may turn out to be unsatisfactory. The natural correct alternative is to use fuzzy mathematics formalisms.

In accordance with this, the purpose of the research is to develop a criterion for the distinguishability of distributions of fuzzy values, free from the shortcomings of the Kullback criterion, in the problem of choosing parameters for identifying the state of systems.

**Development of a criterion for the distinguishability of distributions of fuzzy values.** Let’s consider the standard procedure for using some parameter of an object to diagnose its state. Let an object be in one of two states $H_1$ or $H_2$. Let's introduce membership functions of $(L-R)$ type of fuzzy controlled parameter $x$ for the states $H_1$ and $H_2$:

$$\mu_{H_1}(x)=<m_1,\alpha_1,\beta_1>,\mu_{H_2}(x)=<m_2,\alpha_2,\beta_2>.$$

The simplest special case, when these functions are triangular, is shown in fig. 1.

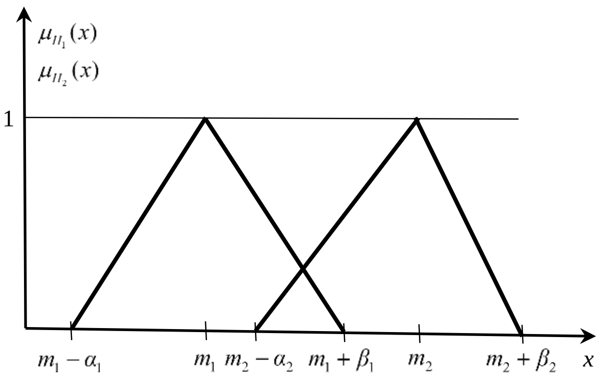


Fig.1. Membership functions $\mu_{H_1}(x), \mu_{H_2}(x)$

In this case, the area of possible observed values of the parameter $x$ is divided into subareas:

$I_1=[m_1-\alpha_1, m_2-\alpha_2]$ – is the subarea of values $x$, that can be observed only if the object is in state $H_1$;

$I_2=[m_1+\beta_1, m_2+\beta_2]$ – is the subarea of values $x$, that can be observed only when the object is in state $H_2$;

$I_{1,2}=[m_2-\alpha_2, m_1+\beta_1]$ – is the subarea of values $x$ that can be observed for both state $H_1$ and state $H_2$ (compatibility interval);

$I_0=[m_1-\alpha_1, m_1+\beta_1]\cup[m_2-\alpha_2, m_2+\beta_2]$ – is the area of all possible observed values of the parameter $x$

Now let's introduce the criterion:

$$\tau=\frac{I_{1,2}}{I_0}. \tag{3}$$

It is easy to see that criterion (3) is free from the shortcomings of the Kullback criterion (1). Indeed, the calculation result of the criterion calculation does not depend on the order of calculation of its components. Further, this criterion is normalized.

The criterion $\tau$ is 0 ($\tau=0$) if the length of the compatibility interval $I_{1,2}$ is equal to 0, that is, the areas of possible values of the parameter $x$ for the states $H_1$ and $H_2$ do not intersect.

The criterion $\tau$ is 1 ($\tau=1$) if the areas of possible values of the parameter $x$ for the states $H_1$ and $H_2$ coincide. Thus, the length of the compatibility interval $I_{1,2}$ in a certain, easily interpreted way, characterizes the informativeness value of the parameter $x$.

At the same time, for describing the uncertainty of the initial data, it is proposed to use membership functions of $(L-R)$ type that are convenient for carrying out calculations. Thus, the task is solved.

It is clear that the level of informativeness of the criterion clearly depends on the length of the compatibility interval $I_{1,2}$. Wherein, if the observed value of the controlled parameter turns out to be within this interval, then this fact, in itself, does not contain any information regarding the state of the object. However, this information can be extracted using analytical descriptions of the membership functions of controlled parameters for different states of the object $H_1$ and $H_2$. The desired effect is achieved as follows.

For each of the membership functions of the values of the controlled parameter, we define its probabilistic counterpart. To this end, we determine the areas under the curves given by the functions $\mu_{H_1}(x)$ and $\mu_{H_2}(x)$.

Let's introduce

$$S_1=\int_{m_1-\alpha_1}^{m_1+\beta_1}\mu_{H_1}(x)dx;$$

$$S_2=\int_{m_2-\alpha_2}^{m_2+\beta_2}\mu_{H_2}(x)dx.$$

Then

$$\hat{\mu}_{H_2}(x)=\frac{\mu_{H_2}(x)}{S_2}; \qquad \hat{\mu}_{H_1}(x)=\frac{\mu_{H_1}(x)}{S_1}. \tag{4}$$

The functions given by formulas (4) have all the properties of the distribution densities of random values [8, 9]: they are nonnegative and

$$\int_{-\infty}^{\infty}\hat{\mu}_{H_1}(x)dx=1;$$

$$\int_{-\infty}^{\infty}\hat{\mu}_{H_2}(x)dx=1.$$

As a result, they can be used to calculate the probabilities that the observed values of the controlled parameter $x$ fall into the interval $I_{1,2}$ for the states $H_1$ and $H_2$:

$$P_{H_1}(x\in I_{1,2})=\int_{m_2-\alpha_2}^{m_1+\beta_1}\hat{\mu}_{H_1}(x)dx; \tag{5}$$

$$P_{H_2}(x\in I_{1,2})=\int_{m_2-\alpha_2}^{m_1+\beta_1}\hat{\mu}_{H_2}(x)dx. \tag{6}$$

Possible differences in the numerical values of the obtained probabilities contain additional information about the state of the object, increasing the information value of the controlled parameter. Let's consider examples.

Example 1. Let's introduce triangular functions of $(L-R)$ type of a fuzzy controlled parameter $x$ for the states $H_1$ and $H_2$ (fig.2):

$$\mu_{H_1}(x)=\begin{cases}0, x<10;\\ \dfrac{x-10}{15}, 10\le x<25;\\ \dfrac{40-x}{15}, 25\le x\le 40;\\ 0, x>40;\end{cases}$$

$$\mu_{H_2}(x)=\begin{cases}0,x\le 30;\\ \dfrac{x-30}{25},30\le x<55;\\ \dfrac{70-x}{15},55\le x\le 70;\\ 0,x>70.\end{cases}$$

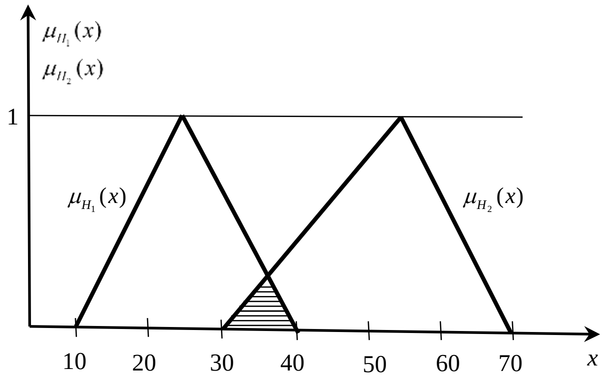


Fig. 2. Graphs $\mu_{H_1}(x)$, $\mu_{H_2}(x)$

The interval $I_{1,2}$ of compatibility of possible values $x$ for $H_1$ and $H_2$ is $I_{1,2}=[30;40]$. Now, using formulas (4)–(6), we calculate the probabilities that the parameter $x$ will fall into $I_{1,2}$ for $H_1$ and $H_2$.

We have

$$S_1=\int_{m_1-\alpha_1}^{m_1+\beta_1}\mu_{H_1}(x)dx=\frac{1}{15}\int_{10}^{25}(x-10)dx+\frac{1}{15}\int_{25}^{40}(40-x)dx=$$

$$=\frac{1}{15}\left[\frac{x^2}{2}\bigg|_{10}^{25}-10x\bigg|_{10}^{25}\right]+\frac{1}{15}\left[40x\bigg|_{25}^{40}-\frac{x^2}{2}\bigg|_{25}^{40}\right]=15;$$

$$S_2=\int_{m_2-\alpha_2}^{m_2+\beta_2}\mu_{H_2}(x)dx=\int_{30}^{55}\frac{x-30}{25}dx+\int_{55}^{70}\frac{70-x}{15}dx=20.$$

$$\hat{\mu}_{H_1}(x)=\frac{\mu_{H_1}(x)}{S_1}=\begin{cases}0,x<10\\ \dfrac{x-10}{15\cdot 15},10\le x<25;\\ \dfrac{40-x}{15\cdot 15},25\le x<40;\\ 0,x>40;\end{cases}$$

$$\hat{\mu}_{H_2}(x)=\frac{\mu_{H_2}(x)}{S_2}=\begin{cases}0,x<30;\\ \dfrac{x-30}{25\cdot 20},10\le x<25;\\ \dfrac{70-x}{15\cdot 20},55\le x\le 70;\\ 0,x>70.\end{cases}$$

Then

$$P_{H_2}(x\in I)=\int_{m_2-\alpha_2}^{m_1+\beta_1}\hat{\mu}_{H_2}(x)dx=\int_{30}^{40}\frac{x-30}{25\cdot 20}dx=$$

$$=\frac{x^2}{2\cdot 25\cdot 20}\bigg|_{30}^{40}-\frac{30x}{25\cdot 20}\bigg|_{30}^{40}=0{,}1.$$

$$P_{H_1}(x\in I)=\int_{m_2-\alpha_2}^{m_1+\beta_1}\hat{\mu}_{H_1}(x)dx=\int_{30}^{40}\frac{40-x}{15\cdot 15}dx=$$

$$=\frac{40x}{15\cdot 15}\bigg|_{30}^{40}-\frac{x^2}{2\cdot 15\cdot 15}\bigg|_{30}^{40}=0{,}222.$$

Example 2. Let's introduce the membership functions of the controlled parameter $x$ for the states $H_1$ and $H_2$ (fig.3):

$$\mu_{H_1}(x)=\begin{cases}0,x<10\\ \dfrac{x-10}{25},10\le x<35;\\ \dfrac{40-x}{5},35\le x\le 40;\\ 0,x>40;\end{cases}$$

$$\mu_{H_2}(x)=\begin{cases}0,x<30;\\ \dfrac{x-30}{35},30\le x<65;\\ \dfrac{70-x}{5},65\le x\le 70;\\ 0,x>70.\end{cases}$$

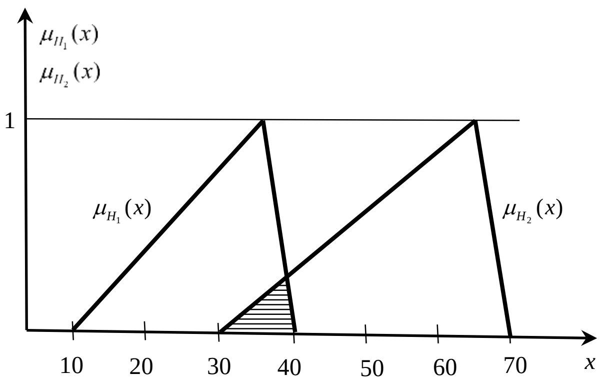


Fig. 3. Graphs $\mu_{H_1}(x)$, $\mu_{H_2}(x)$

The compatibility interval of possible values $x$ for $H_1$ and $H_2$ is $I=[30;40]$, that is, it coincides with this interval in example 1. Further we have:

$$S_1=\int_{m_1-\alpha_1}^{m_1+\beta_1}\mu_{H_1}(x)dx=\frac{1}{25}\int_{10}^{35}\frac{x-10}{25}dx+\frac{1}{5}\int_{35}^{40}\frac{40-x}{5}dx=$$

$$=\frac{1}{25}\left[\frac{x^2}{2}\bigg|_{10}^{35}-10x\bigg|_{10}^{35}\right]+\frac{1}{5}\left[40x\bigg|_{35}^{40}-\frac{x^2}{2}\bigg|_{35}^{40}\right]=15;$$

$$S_2 = \int_{m_2-\alpha_2}^{m_2+\beta_2} \mu_{H_2}(x)dx = \frac{1}{35}\int_{30}^{65}\frac{x-30}{35}dx + \frac{1}{5}\int_{65}^{70}\frac{70-x}{5}dx = 20.$$

$$\hat{\mu}_{H_1}(x) = \frac{\mu_{H_1}(x)}{S_1} = \begin{cases} 0, x<10 \\ \dfrac{x-10}{25\cdot 15}, 10 \le x < 35; \\ \dfrac{40-x}{5\cdot 15}, 35 \le x \le 40; \\ 0, x>40; \end{cases}$$

Then

$$\hat{\mu}_{H_2}(x) = \frac{\mu_{H_2}(x)}{S_2} = \begin{cases} 0, x<30; \\ \dfrac{x-30}{35\cdot 20}, 30 \le x < 65; \\ \dfrac{70-x}{5\cdot 20}, 65 \le x \le 70; \\ 0, x>70. \end{cases}$$

It follows that

$$P_{H_1}(x \in I) = \int_{m_2-\alpha_2}^{m_1+\beta_1} \hat{\mu}_{H_1}(x)dx = \int_{30}^{40}\frac{40-x}{5\cdot 15}dx =$$

$$= \frac{40x}{5\cdot 15}\Big|_{30}^{40} - \frac{x^2}{2\cdot 5\cdot 15}\Big|_{30}^{40} = 0,67.$$

$$P_{H_2}(x \in I) = \int_{m_2-\alpha_2}^{m_1+\beta_1} \hat{\mu}_{H_2}(x)dx = \int_{30}^{40}\frac{x-30}{35\cdot 20}dx =$$

$$= \frac{x^2}{2\cdot 35\cdot 20}\Big|_{30}^{40} - \frac{30x}{35\cdot 20}\Big|_{30}^{40} = 0,071.$$

Example 3. Let's introduce the membership functions of the controlled parameter $x$ for the states $H_1$ and $H_2$ (fig.4):

$$\mu_{H_1}(x) = \begin{cases} 0, x<10 \\ \dfrac{x-10}{5}, 10 \le x < 15; \\ \dfrac{40-x}{25}, 15 \le x \le 40; \\ 0, x>40. \end{cases}$$

$$\mu_{H_2}(x) = \begin{cases} 0, x<30; \\ \dfrac{x-30}{5}, 30 \le x < 35; \\ \dfrac{70-x}{35}, 35 \le x \le 70; \\ 0, x>70. \end{cases}$$

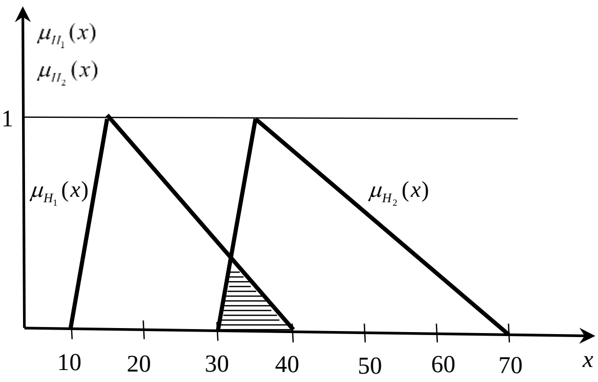


Fig. 4. Graphs $\mu_{H_1}(x)$, $\mu_{H_2}(x)$

The compatibility interval of possible values $x$ for $H_1$ and $H_2$ is still $I = [30; 40]$. Further, similarly to considered above, we have:

$$S_2 = \int_{m_2-\alpha_2}^{m_2+\beta_2} \mu_{H_1}(x)dx = 20;$$

$$S_2 = \int_{m_2-\alpha_2}^{m_2+\beta_2} \mu_{H_2}(x)dx = 20.$$

Then

$$\hat{\mu}_{H_1}(x) = \frac{\mu_{H_1}(x)}{S_1} = \begin{cases} 0, x<10 \\ \dfrac{x-10}{5\cdot 15}, 10 \le x < 15; \\ \dfrac{40-x}{25\cdot 15}, 15 \le x \le 40; \\ 0, x>40; \end{cases}$$

$$\hat{\mu}_{H_2}(x) = \frac{\mu_{H_2}(x)}{S_2} = \begin{cases} 0, x<30; \\ \dfrac{x-30}{5\cdot 20}, 30 \le x < 35; \\ \dfrac{70-x}{35\cdot 20}, 35 \le x \le 70; \\ 0, x>70. \end{cases}$$

It follows that

$$P_{H_2}(x \in I) = \int_{m_2-\alpha_2}^{m_1+\beta_1} \hat{\mu}_{H_2}(x)dx = \int_{30}^{35}\frac{x-30}{5\cdot 20}dx +$$

$$+\int_{355}^{40}\frac{70-x}{35\cdot 20}dx == \frac{x^2}{2\cdot 5\cdot 20}\Big|_{30}^{35} - \frac{30x}{5\cdot 20}\Big|_{30}^{35} +$$

$$+\frac{70x}{35\cdot 20}\Big|_{35}^{40} - \frac{x^2}{2\cdot 55\cdot 20}\Big|_{35}^{40} = 0,125 = 0,457;$$

$$P_{H_1}(x \in I) = \int_{m_2-\alpha_2}^{m_1+\beta_1} \hat{\mu}_{H_1}(x)dx = \int_{30}^{40}\frac{40-x}{25\cdot 15}dx =$$

$$= \frac{40x}{25\cdot 15}\Big|_{30}^{40} - \frac{x^2}{2\cdot 25\cdot 15}\Big|_{30}^{40} = 0,133.$$

Let's analyze the results obtained in the examples.

The calculated values of the probabilities of falling the controlled parameter $x$ into the compatibility interval $I_{1,2}$ for different states of the object $H_1$ and $H_2$ obviously depend on the analytical description of the membership functions $\hat{\mu}_{H_1}(x),\ \hat{\mu}_{H_2}(x)$.

In the given examples, the membership functions of the fuzzy parameter $x$ are deliberately chosen so that their carriers for the states $H_1$ and $H_2$ coincide, but their modal values are different. Comparison of the calculated values of the probabilities of the controlled parameter $x$ falling into the compatibility interval $I_{1,2}$ for $H_1$ and $H_2$ shows the existence of their significant differences.

Thus, a possible suggestive idea that the level of informativeness of the controlled parameter depends only on the length of the interval of compatibility of membership functions for different states $H_1$ and $H_2$ is not quite accurate. The use of analytical descriptions of these membership functions and the associated possibility of calculating the probabilities of a controlled parameter falling into the compatibility interval can significantly increase its informativeness. This circumstance is especially important in cases where the length of the compatibility interval is large.

Now it can be noted that the results of the carried out research for the case when the set of possible states of the object contains only two states can easily be extended to the general case.

Let, for example, in the task of assessing the quality of an object, the following states be possible: $H_1$ (low), $H_2$ (satisfactory), $H_3$ (good), $H_4$ (excellent).

At the same time, for some controlled parameter $x$, the membership functions $\mu_{H_k}(x),\ k=1,2,3,4,$ of values of this parameter for each of the states are introduced (fig. 5).

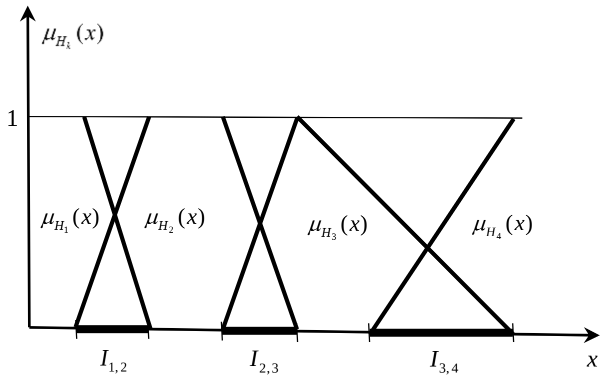


Fig. 5. Graphs of membership functions of a controlled parameter $x$ in intervals of possible values

It is naturally that the compatibility intervals $I_{1,2}$, $I_{2,3}$, $I_{3,4}$ appear.

For each interval $I_{k,k+1}$ let's calculate the probabilities of the controlled parameter falling into this interval for the states of the object $H_k$ and $H_{k+1}$, $k=1,2,3$,

$$\begin{aligned}
&P_{H_k}(x)=\int_{m_{k+1}-\alpha_{k+1}}^{m_k+\beta_k}\hat{\mu}_{H_k}(x)dx;\\
&P_{H_{k+1}}(x)=\int_{m_{k+1}-\alpha_{k+1}}^{m_k+\beta_k}\hat{\mu}_{H_{k+1}}(x)dx;\\
&\hat{\mu}_{H_k}(x)=\frac{\mu_{H_k}(x)}{S_k};\\
&S_k=\int_{m_k-\alpha_k}^{m_k+\beta_k}\mu_{H_k}(x)dx,\ k=1,2.
\end{aligned}\qquad(7)$$

Possible differences in the values of these probabilities improve the accuracy of identifying the state of the object.

One more step can be taken in this direction. Let for the measured value $x_\varsigma$ of the controlled parameter $x$, an interval $[x_\varsigma-\frac{\Delta}{2}, x_\varsigma+\frac{\Delta}{2}]$ be introduced that covers the true value of this parameter with a given probability $P$. Let's calculate the probability of the parameter value falling into this interval for each of the possible states $H_k$:

$$\hat{P}_{H_k}(x)=\int_{x_\varsigma-\frac{\Delta}{2}}^{x_\varsigma+\frac{\Delta}{2}}\mu_{H_k}(x)dx=\mu_{H_k}(x_\varsigma)\cdot\Delta,\ k=1,2.\qquad(8)$$

The results of calculating these probabilities for different $H_k$ contain important information about the state of the object. Let's consider the corresponding method.

Let's introduce:

an event $A$ – is a fall of the value of the controlled parameter into the compatibility interval;

$P_{H_k}(x)$ – is the probability that the parameter value will fall into the compatibility interval, provided that the object is in state $H_k, k=1,2$.

Since

$$P(A_1H_k)=P(A_1/H_k)\cdot P(H_k)=P(H_k/A)\cdot P(A),$$

then

$$P(H_k/A)=\frac{P(A/H_k)\cdot P(H_k)}{P(A)}.\qquad(9)$$

At the same time, since

$$P(A)=P(A/H_1)\cdot P(H_1)+P(A/H_2)\cdot P(H_2)\qquad(10)$$

then, substituting (10) into (9), we obtain:

$$P(H_k/A)=\frac{P(A/H_k)\cdot P(H_k)}{\sum_{k=1}^{2}P(A/H_k)\cdot P(H_k)},\qquad(11)$$

which corresponds to the Bayesian theorem [10, 11].

Thus, relations (8)–(11) provide the possibility of constructive use of analytical descriptions of membership functions of fuzzy values of the controlled parameter from the compatibility interval to identify the probability distributions of the states of the monitored object. At the same time, it is clear that any refinement of the description of these membership functions creates the prerequisites for increasing the informativeness of the corresponding controlled parameters. Let's show that.

Let's return to the considered above problem of assessing the informativeness of the controlled parameter $x$, which is used to identify the states of the object $H_1$ and $H_2$.

Let, as before, the corresponding membership functions of $(L-R)$ type have the form $\mu_{H_1}(x) = <\mu_1, \alpha_1, \beta_1>$, $\mu_{H_2}(x) = <\mu_2, \alpha_2, \beta_2>$.

Let's now assume that, based on the results of processing the initial data, the fuzziness of the parameter $m_1$ of the odd value $x$ for the state $H_1$ is established and the corresponding membership function is determined as follows $\mu_{H_1}(m_1) = <m_{m_1}, \alpha_{m_1}, \beta_{m_1}>$.

Let's analyze the situation. To this end, we recalculate the probability that the value $x$ falls into the compatibility interval for the state $H_1$. We have

$$S_1 = \int_{m_{m_1}-\alpha_{m_1}}^{m_{m_1}+\beta_{m_1}} \left[ \int_{m_1-\alpha_1}^{m_1+\beta_1} \mu_{H_1}(x)dx \right] \mu_{H_1}(m_1)dm_1 =$$
$$= \int_{m_{m_1}-\alpha_{m_1}}^{m_{m_1}} \left[ \int_{m_1-\alpha_1}^{m_1} \mu_{H_1}(x)dx + \int_{m_1}^{m_1+\beta_1} \mu_{H_1}(x)dx \right] \mu_{H_1}(m_1)dm_1 + \quad (12)$$
$$+ \int_{m_{m_1}}^{m_{m_1}-\beta_{m_1}} \left[ \int_{m_1-\alpha_1}^{m_1} \mu_{H_1}(x)dx + \int_{m_1}^{m_1+\beta_1} \mu_{H_1}(x)dx \right] \mu_{H_1}(m_1)dm_1;$$

$$\hat{\mu}_{H_1}(x) = \frac{1}{S_1}\mu_{H_1}(x); \quad (13)$$

$$P_{H_1}(x \in I_{1,2}) = \int_{m_{m_1}-\alpha_{m_1}}^{m_{m_1}} \left[ \int_{m_2-\alpha_2}^{m_1+\beta_1} \hat{\mu}_{H_1}(x)dx \right] \hat{\mu}_{H_1}(m_1)dm_1 +$$
$$+ \int_{m_{m_1}}^{m_{m_1}+\beta_{m_1}} \left[ \int_{m_2-\alpha_2}^{m_1+\beta_1} \hat{\mu}_{H_1}(x)dx \right] \hat{\mu}_{H_1}(m_1)dm_1. \quad (14)$$

Let's perform calculations using formulas (12)–(14), choosing the initial data for $\mu_H(x)$ from Example 1, that is, $\mu_{H_1}(x) = <25; 15; 15>$, and adding $\mu_{H_1}(m) =< 25; 2; 10>$. As a result, we get $P_{H_1}(x \in I_{1,2}) = 0,61$ (instead of the previous 0.222). Thus, the shift to the right of the value of $m_1$ expectedly has led to a noticeable increase in the numerical value of the probability $P_{H_1}(x \in I_{1,2})$, while increasing the informativeness of parameter $x$.

Finally, it should be noted that some additional contribution to the assessment of the informativeness of a controlled parameter can be made by differences in the level of membership functions of the observed value of this parameter for different states of the system.

The direction of further research is the assessment of the informativeness of the controlled parameters in a situation where they are used to evaluate the effectiveness of the system in a multicriteria problem. A possible approach is proposed in [12].

**Conclusions.** A method for identifying the state of systems under conditions of fuzzy initial data has been developed.

A symmetrical criterion for evaluating the informativeness of the controlled parameters of the system, the values of which are not clearly specified, is proposed. The situation is considered when the parameters of membership functions of a fuzzy controlled parameter are themselves fuzzy. A method for solving the binetch problem that arises in this case is proposed.

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*Received 05.09.2022*

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