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SYNTHESIS OF DESIGN PARAMETERS OF MULTI-PURPOSE DYNAMIC SYSTEMS

Two problems related to the optimization of linear stationary dynamic systems are considered. A general formulation of the multi-purpose problem of optimal control with the choice of design parameters is given. As a special case, the problem of multi-objective optimization of a linear system according to an integral quadratic criterion with a given random distribution of initial deviations is considered. The solution is based on the method of simultaneously reducing two positive-definite quadratic forms to diagonal form. Analytical results have been obtained that make it possible to calculate the mathematical expectation of the criterion under the normal multidimensional distribution law of the vector of random initial perturbations. The inverse problem of stability theory is formulated: to find a vector of structural parameters that ensure the stability of the system and a given average value of the quadratic integral quality criterion on a set of initial perturbations. The solution of the problem is proposed to be carried out in two stages. The first stage involves deriving a general solution to the Lyapunov matrix equation in terms of the elements of the system matrix. To achieve this, the state space is mapped onto the eigen-subspace of the positive-definite matrix corresponding to the integral quadratic performance criterion. It has been established that this solution is determined by an arbitrary skew-symmetric matrix or by the corresponding set of arbitrary constants. In contrast, when the system matrix depends linearly on the vector of design parameters, a linear system of equations can be formulated with respect to the unknown parameters and arbitrary constants present in the general solution of the inverse stability problem. In general, such a system is consistent and admits an infinite number of solutions that satisfy the initial requirements for the elements of the symmetric matrices in the Lyapunov.

Keywords: stability, integral quadratic functional, Lyapunov matrix equation, inverse stability problem, multi-purpose dynamic systems, linear stationary systems, parametric optimization.

1. Introduction. The problem of optimizing the parameters of stable dynamic systems based on integral quadratic functional (IQF) of transient processes has been studied and solved in various theoretical and applied contexts. Fundamental results in this area were obtained in [1, 2, 3]. These results have been regarded as classical for a long time.

They differ somewhat in the formulation of the problem of optimal control for dynamic systems combined with the selection of design parameters. The mathematical formulation of the optimal control problem with parameters based on the maximum principle is addressed in [4, 5]. The issue of parametric optimization of the IQF with a known statistical distribution of initial perturbations is discussed in [3]. This work also formulates and proposes a solution to the problem of selecting the elements of the matrix of a linear stationary dynamic system that optimize the mathematical expectation of the IQF. The general solution to this problem is expressed in terms of matrix algebraic equations involving several parameters and a set of auxiliary variables. Analyzing the entire spectrum of research in the field of theory and practice of ensuring the stability of dynamic systems, the field can be divided into two primary classes: problems of analysis and problems of synthesis of stable systems, corresponding to the direct and inverse problems of stability theory. Direct problems focus on determining whether a system with specified parameters is stable. In contrast, inverse problems aim to identify a vector of parameters that ensures the stability of the system to be constructed.

Among the methods for solving inverse problems of stability theory, two fundamental directions can be distinguished. The first direction is related to the control of the distributed roots of the characteristic equation of the system of differential equations. These methods are indirectly related to the control of the coefficients of the characteristic equation by selecting the appropriate design parameters [6]. The theory of modal control is closely related to this direction [7]. Information on the distribution of roots also makes it possible to find indirect characteristics of transient processes, such as the degree of stability and the degree of oscillation of the synthesized system but does not make it possible to calculate direct indicators: the time of the transient process and the degree of oscillation.

The second direction gained significant momentum in the second half of the last century and is associated with the works [8-10], particularly in the context of the Lyapunov matrix equation, as well as related studies [11,12] on IQF. The direct stability problem, based on Lyapunov equations, involves evaluating the solutions for given parameters of the dynamic system matrix. The inverse problem can be formulated as the task of finding the parameters of the dynamic system matrix that satisfy the Lyapunov matrix equation.

2. Formulation of a parametric multi-criteria optimal control problem. Among the variety of formulations of optimal control problems for continuous dynamic systems, there exists an important problem that has been formulated and solved in general form. Let us briefly outline the mathematical formulation. A stationary dynamic system of the following form is given:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{\alpha}), \tag{1}$$

where: $\boldsymbol{x} \in X$ – is the state vector of the system, dim $X = n$; $\boldsymbol{u} \in U$ – vector of control parameters, dim$U = m$; $\boldsymbol{\alpha} \in A$ – vector of design parameters, dim $A = s$. Vector function $\boldsymbol{f}$ and its partial derivatives $\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}'}$, $\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\alpha}}$ are assumed to be continuous at $\boldsymbol{u} \in U$ and any $\boldsymbol{x}$, $\boldsymbol{\alpha}$. The control objective is defined by a target vector

$$\boldsymbol{c} = (\boldsymbol{x}_0, \boldsymbol{x}_1, \tau), \tag{2}$$

where $\boldsymbol{x}_0 \in X$ – is the vector of the initial state, $\boldsymbol{x}_1 \in X$ – the vector of the final state, τ – is the time of transition of the system from $\boldsymbol{x}_0$ to $\boldsymbol{x}_1$. The quality criterion of the controlled process is expressed as

$$J(\boldsymbol{c},\boldsymbol{u},\boldsymbol{\alpha}) = \int_0^{\tau} f_0(\boldsymbol{x},\boldsymbol{u},\boldsymbol{\alpha})dt. \tag{3}$$

The problem of optimal control of a system (1) with variable parameters is formulated as follows: for a given system objective, find the constant vector $\boldsymbol{\alpha}$ and the function $\boldsymbol{u}$(t) that minimizes the quality functional (3). In works [4, 5], necessary conditions for solving the formulated problem were obtained based on the maximum principle. It follows from these conditions that the optimal functions $\boldsymbol{\alpha}$ and $\boldsymbol{u}(t)$ are functions of the vector $\boldsymbol{c}$, and the explicit form of these functions in general is unknown.

Let the system being constructed be multi-criteria, i.e., a subset is given:

$$C \subseteq X \times X \times T, \quad T \in (0,\infty) \tag{4}$$

defines the set of objectives of the form (2) that can be a goal for the control system and let a probability measure $p(\boldsymbol{c})$ be defined on the subset C that defines the probability of achieving the given objective. It would be natural to pose the following parametric optimization problem for the multi-objective system (1) and a set of objectives (4), find a parameter $\boldsymbol{\alpha}$ vector that minimizes criterion (3) at each point of the set C. In general, such a formulation of the problem is not correct, since the vector of parameters that is optimal for one of the goals of the system will not be optimal for another. In this regard, we will optimize a certain average criterion for all permissible goals of the system. Let's introduce the function:

$$\Psi(\boldsymbol{c},\boldsymbol{\alpha}) = \min_{\boldsymbol{u}\in U} J(\boldsymbol{c},\boldsymbol{u},\boldsymbol{\alpha}) \tag{5}$$

which is the minimum value of criterion (3) with a fixed vector of constructive parameters $\boldsymbol{\alpha}$ and a fixed target vector $\boldsymbol{c}$. Let's find the mathematical expectation of function (5):

$$I(\boldsymbol{\alpha}) = \int_C p(\boldsymbol{c})\Psi(\boldsymbol{c},\boldsymbol{\alpha})dC, \tag{6}$$

determining the average quality of the system with a fixed parameter vector.

Now the formulation of the parametric optimization problem can be proposed as follows: for the system (1), the set of goals (4) and criterion (3), choose a vector of design parameters $\boldsymbol{\alpha}$ that minimizes the average quality (6). The vector of design parameters determined in this way provides optimization of criterion (3) on average for a set of system goals. In many important practical cases, the system is designed for a single use. In these cases, the criterion of the form (6) loses its physical meaning, since it presupposes the repeated use of the system. In this case, it is proposed to optimize the worst conditions of the system's functioning, i.e., instead of functional (6), to optimize the functional

$$I(\boldsymbol{\alpha}) = \max_{\boldsymbol{c}\in C} p(\boldsymbol{c})\Psi(\boldsymbol{c},\boldsymbol{\alpha}).$$

The formulation of the optimization problem in this case is similar to the previous one. In the implementation of the above-mentioned formulation of the problem, the main difficulty is to determine the function $\Psi(\boldsymbol{c},\boldsymbol{\alpha})$ for the finding of which at each point $(\boldsymbol{c},\boldsymbol{\alpha})$ it is necessary to solve the problem of optimal control, which in most cases does not have an analytical solution. However, in many cases, and particularly the problem of optimal stabilization, it is possible to obtain an analytic dependence $\Psi(\boldsymbol{c},\boldsymbol{\alpha})$

3. Multi-purpose stabilization of a linear dynamic system. Let us consider a dynamic system of the form

$$\dot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{\alpha})\boldsymbol{x}, \tag{7}$$

where $\boldsymbol{F}(\boldsymbol{\alpha}) - n \times n$ is the matrix-function of the s-dimensional vector $\boldsymbol{\alpha}$ of the constructive parameters. Let the control objective be defined as

$$\boldsymbol{c} = (\boldsymbol{x}_0 \in X, 0, \tau = \infty). \tag{8}$$

In other words, a set of stabilization problems under arbitrary initial conditions and infinite time is considered. The perturbed motion of the system caused by the deviation $\boldsymbol{x}_0$ from the zero-equilibrium point will be evaluated using the IQF.

$$I = \int_0^{\infty} \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{x}(t)dt, \tag{9}$$

where $\boldsymbol{Q}$ – given positive symmetric matrix. In this case [8] the value of the criterion I is in the form of $I = \boldsymbol{x}_0^{\mathrm{T}}\boldsymbol{S}\boldsymbol{x}_0$, where $\boldsymbol{S}$ – positive solution of the Lyapunov matrix equation

$$\boldsymbol{F}^{\mathrm{T}}(\boldsymbol{\alpha})\boldsymbol{S} + \boldsymbol{S}\boldsymbol{F}(\boldsymbol{\alpha}) + \boldsymbol{Q} = \boldsymbol{0}. \tag{10}$$

It should be noted [13], that the positivity of the matrix $\boldsymbol{S}$ for arbitrarily positive matrix $\boldsymbol{Q}$ is a necessary and sufficient condition for the matrix $\boldsymbol{F}(\boldsymbol{\alpha})$ in the context of Hurwitzness criteria.

Let the initial perturbations $\boldsymbol{x}_0$ be distributed according to a normal law with zero mathematical expectation and a covariance matrix $\boldsymbol{P}$:

$$p(\boldsymbol{x_0}) = \frac{1}{(2\pi)^{n/2}\sqrt{|\boldsymbol{P}|}} e^{-\frac{1}{2}(\boldsymbol{x}_0)^{\mathrm{T}} \boldsymbol{P}^{-1}\boldsymbol{x}_0}. \quad (11)$$

It is known [13], that there exists a coordinate transformation $\boldsymbol{z} = \boldsymbol{Hx}$, such that the quadratic forms in $I = \boldsymbol{x}_0^{\mathrm{T}}\boldsymbol{S}\boldsymbol{x}_0$ and $V = \boldsymbol{x}_0^{\mathrm{T}}\boldsymbol{P}^{-1}\boldsymbol{x}_0$ the coordinate system $\boldsymbol{z}_1$, $\boldsymbol{z}_2,\ldots,\boldsymbol{z}_n$ will have a diagonal form $I = \boldsymbol{z}_0^{\mathrm{T}}\boldsymbol{\Lambda}\boldsymbol{z}_0$, $V = \boldsymbol{z}_0^{\mathrm{T}}\boldsymbol{z}_0$, where $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_k)$; λ_k – solutions of the characteristic equation of a bundle of quadratic forms $(I,\ V)$ [13]:

$$\left|\boldsymbol{S} - \lambda\boldsymbol{P}^{-1}\right| = 0. \quad (12)$$

In coordinates $\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_n$ the mathematical expectation of the IQF on the set of initial perturbations with distribution (11) will take the form

$$\bar{J} = k\int_{-\infty}^{\infty} e^{-\frac{1}{2}\sum_{k=1}^{n}(z_{0k})^2} \sum_{j=1}^{n}\lambda_j (z_{0j})^2 \, dz_{0,1} dz_{0,2} \ldots dz_{0,n}, \quad (13)$$

where $k = \dfrac{1}{(2\pi)^{n/2}|\boldsymbol{P}|^{n/2}}$.

The ratio (13) is equivalent to the following:

$$\bar{J} = k\sum_{j=1}^{n}\lambda_j \int_{-\infty}^{\infty} z_{0j}^2 e^{-\frac{1}{2}\sum_{k=1}^{n}(z_{0k})^2} dz_{0,1} dz_{0,2}, \ldots, dz_{0,n}. \quad (14)$$

It is not difficult to show that the integral in (14) can be calculated by the formula $(2\pi)^{\frac{n}{2}}$, which after substituting in (14), will give the following result:

$$\bar{J} = |\boldsymbol{P}|^{-n/2} \sum_{j=1}^{n}\lambda_j. \quad (15)$$

Ratio (12) can be written in the equivalent form

$$|\boldsymbol{PS} - \lambda\boldsymbol{E}| = 0, \quad (16)$$

from which it follows that the numbers $\lambda_1,\ \lambda_2, \ldots,\ \lambda_n$ coincide with the eigenvalues of the matrix $\boldsymbol{PS}$. Therefore, the average values of the IQF on the set of random initial perturbations (15) will take the form

$$\bar{J} = |\boldsymbol{P}|^{n/2}\ \mathrm{tr}(\boldsymbol{PS}). \quad (17)$$

Thus, the problem of optimal stabilization of a linear dynamical system over the mean value of the IQF on a set of initial states distributed according to a normal law can be formulated as follows: minimize the criterion (17) at $\boldsymbol{S}$, satisfying equation (10) and the given dependencies for $\boldsymbol{F}(\boldsymbol{\alpha})$ where $\boldsymbol{\alpha} \in A$. It is not difficult to see that the solution of such an optimization problem in a general form is impossible and quite difficult in numerical formulation. In this regard, let us consider a slightly different approach to solving the problem of stabilization based on the inverse problem of the theory of stability.

4. Inverse problem of stability theory. Let two symmetric positive-definite matrices $\boldsymbol{Q}$ and $\boldsymbol{S}$ be given, which, as before, represent the matrices of the IQF and the solution to the Lyapunov equations

$$\boldsymbol{F}^{\mathrm{T}}\boldsymbol{S} + \boldsymbol{SF} + \boldsymbol{Q} = \boldsymbol{0}. \quad (18)$$

It is necessary to find a solution to equation (18) with in terms of the matrix $\boldsymbol{F}$. Since the matrix equation (18) is equivalent to a system $\dfrac{n(n+1)}{2}$ of linear equations for n^2 unknowns, such a system will generally be compatible and have an infinite number of solutions. To find these solutions, we will proceed as follows. Find the coordinate transformation

$$\boldsymbol{y} = \boldsymbol{R}\,\boldsymbol{x} \quad (19)$$

such that the matrices $\boldsymbol{S}$ and $\boldsymbol{Q}$ simultaneously take a diagonal form $\boldsymbol{E}$ and $\boldsymbol{\Lambda}$, where $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$, $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the solutions of the equation $|\boldsymbol{Q} - \lambda\boldsymbol{S}| = 0$.

In [13] the algorithm for finding the transformation (19) is given. Since the matrix $\boldsymbol{R}$ satisfies the ratio:

$$\boldsymbol{R}^{\mathrm{T}}\boldsymbol{SR} = \boldsymbol{E}, \quad (20)$$

then from (20) it follows

$$\boldsymbol{S} = (\boldsymbol{R}^{\mathrm{T}})^{-1}\boldsymbol{R}^{-1}. \quad (21)$$

Substituting (21) in (18), we get

$$\boldsymbol{F}^{\mathrm{T}}(\boldsymbol{R}^{\mathrm{T}})^{-1}\boldsymbol{R}^{-1} + (\boldsymbol{R}^{\mathrm{T}})^{-1}\boldsymbol{R}^{-1}\boldsymbol{F} + \boldsymbol{Q} = \boldsymbol{0}. \quad (22)$$

Multiplying (22) on the left by $\boldsymbol{R}^{\mathrm{T}}$ and right by $\boldsymbol{R}$, we get

$$\bar{\boldsymbol{F}}^{\mathrm{T}} + \bar{\boldsymbol{F}} = -\boldsymbol{\Lambda}, \quad (23)$$

where $\bar{\boldsymbol{F}} = \boldsymbol{R}^{-1}\boldsymbol{FR}$. According to the general theory of solving systems of linear equations, the solution (23) can be represented as $\bar{\boldsymbol{F}} = \bar{\boldsymbol{F}}_1 + \bar{\boldsymbol{F}}_2$, where $\bar{\boldsymbol{F}}_1$ – partial solution of the inhomogeneous system (23), and $\bar{\boldsymbol{F}}_2$ – general solution of the corresponding homogeneous system

$$\bar{\boldsymbol{F}}^{\mathrm{T}} + \bar{\boldsymbol{F}} = \boldsymbol{0}. \quad (24)$$

Let's present an arbitrary skew-symmetric matrix $\boldsymbol{K}$, as a linear combination of basic skew-symmetric matrices $\boldsymbol{K}_{ij}$, consisting of zeros, except for the elements $k_{ij} = -k_{ji} = 1$. Then the matrix $\boldsymbol{K}$ will take the form $\boldsymbol{K} = \sum_{i,j=1, i<j}^{n} c_{ij}\boldsymbol{K}_{ij}$, where $c_{ij} = \dfrac{n(n-1)}{2}$, are arbitrary constants.

From (23) it follows directly that $\bar{\boldsymbol{F}}_1 = -\dfrac{1}{2}\boldsymbol{\Lambda}$, and from (24) it follows that $\bar{\boldsymbol{F}}_2 = \boldsymbol{K}$. Thus

$$\bar{\boldsymbol{F}} = -\frac{1}{2}\boldsymbol{\Lambda} + \boldsymbol{K}. \quad (25)$$

Let's move on to the initial basis by converting the inverse to (24)

$$\boldsymbol{F} = \boldsymbol{R}\bar{\boldsymbol{F}}\boldsymbol{R}^{-1}. \quad (26)$$

After substituting (26) in (25), we get the final form of the set of solutions of Lyapunov's equations in terms of the matrix $\boldsymbol{F}$:

$$\boldsymbol{F} = -\frac{1}{2}\boldsymbol{R\Lambda R}^{-1} + \boldsymbol{RKR}^{-1}. \quad (27)$$

As a partial solution of equation (18), we can also take the matrix

$$\boldsymbol{F} = -\frac{1}{2}\boldsymbol{S}^{-1}\boldsymbol{Q}, \quad (28)$$

which is not difficult to verify by the direct substitution of (28) in (18). It is also evident that the general solution corresponding to (18) of a homogeneous system has the form $\boldsymbol{F_2} = \boldsymbol{S}^{-1}\boldsymbol{K}$.

Thus, the general solution of the Lyapunov equation in terms of the matrix $\boldsymbol{F}$ can be written as

$$\boldsymbol{F} = \boldsymbol{S}^{-1}\left(-\frac{1}{2}\boldsymbol{Q} + \boldsymbol{K}\right). \quad (29)$$

Finally, (29) let us present it in the form of $\boldsymbol{F} = -\frac{1}{2}\boldsymbol{S}^{-1}\boldsymbol{Q} + \sum_{(i,j)} c_{ij}\boldsymbol{F}_{ij}$, where $\boldsymbol{F}_{ij} = \boldsymbol{S}^{-1}\boldsymbol{K}_{ij}$.

Now the problem of choosing a vector of parameters $\boldsymbol{\alpha}$ of a dynamical system that provides the specified dynamic characteristics based on the IQF is reduced to solving the system of equation (30) in terms of the $\boldsymbol{\alpha}$ and c_{ij} :

$$\boldsymbol{F}(\boldsymbol{\alpha}) = \boldsymbol{F}_1 + \sum_{(i,j)} c_{ij}\boldsymbol{F}_{ij}. \quad (30)$$

Confine ourselves to the case of a linear matrix $\boldsymbol{F}(\boldsymbol{\alpha})$ in terms of parameters $\boldsymbol{\alpha} : \boldsymbol{F}(\boldsymbol{\alpha}) = \boldsymbol{F}^0 + \sum_{i=1}^{m} \alpha_i \boldsymbol{F}^i$, where $\boldsymbol{F}^0, \boldsymbol{F}^1, \ldots, \boldsymbol{F}^m$ are the given fixed matrices. In this case, the problem of synthesis of a stable system under consideration is reduced to the solution of a linear system of equations:

$$\sum_{i=1}^{m} \alpha_i \boldsymbol{F}^i - \sum_{(i,j)} c_{ij}\boldsymbol{F}_{ij} = \boldsymbol{F}_1 - \boldsymbol{F}^0. \quad (31)$$

System (31) can be represented as a system n^2 of linear equations in context of $m + \frac{n(n-1)}{2}$ unknown a_i and c_{ij}. The condition for the compatibility of such a system is the condition $m + \frac{n(n-1)}{2} \geq n^2$. The number of unknowns must be not less than the number of equations. The last inequality can be rewritten as

$$m \geq \frac{n(n-1)}{2}. \quad (32)$$

In the general case, let us assume that instead of (32) there is a strict inequality. Let us also assume that the system of matrices $\boldsymbol{F}_{ij}$ and $\boldsymbol{F}^i$, included in (31), has a maximum rank. Let's vectorize the matrices in (31). As a result, we get a linear system of the equation

$$\boldsymbol{Az} = \boldsymbol{b}, \quad (33)$$

where $\boldsymbol{z}^{\mathrm{T}} = (\alpha_{1m}, \alpha_{m1}, c_{12}, \ldots, c_{(n-1)n})$ – vector of unknown constants, and the matrix $\boldsymbol{A}$ and vectors are constructed from the elements of the matrices $\boldsymbol{F}^i$, $\boldsymbol{F}_{ij}$ and $\boldsymbol{F}^0$ in accordance with the matrix equation (31). Since the matrix $\boldsymbol{A}$ is rectangular, of $n^2 \times \left(m + \frac{n(n-1)}{2}\right)$ dimensions, then, in accordance with the general theory of systems of linear equations, the general solution of the system of equations (33) can be found in the form:

$$\boldsymbol{z} = \boldsymbol{z}_0 + \sum_{i=1}^{r} \xi_i \boldsymbol{z}_i, \quad r = \frac{n(n+1)}{2} - m, \quad (34)$$

where $\boldsymbol{z}_0$ – partial solution of the system (33). $\boldsymbol{z}_i$ – linearly independent solutions corresponding to a homogeneous system, ξ_i – arbitrary constants.

The ratio (34) sets a set of parameters of the system $\boldsymbol{\alpha}$ that ensure stability (7). To select specific values $\alpha_1, \alpha_2, \ldots, \alpha_m$ it is necessary to consider the system of restrictions on their values, but this issue is not the subject of investigation in this work.

Conclusions. The paper presents a general formulation of the multi-objective optimal control problem with the selection of design parameters for the controlled system. As a special case, the problem of selecting the average value of quality is formulated based on the integral quadratic functional over a set of initial deviations from the zero equilibrium position, distributed according to a normal distribution.

The inverse stability problem for linear dynamic systems is considered, which involves finding a set of system matrices that satisfy the Lyapunov equation, given the matrices of intensive quadratic forms that appear in the equation. A general expression for the set of the sought stable matrices of the dynamic system is derived.

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СИНТЕЗ КОНСТРУКТИВНИХ ПАРАМЕТРІВ БАГАТОЦІЛЬОВИХ ДИНАМІЧНИХ СИСТЕМ

Розглядаються дві задачі, пов’язані з оптимізацією лінійних стаціонарних динамічних систем. Запропоновано загальну постановку багатоцільової задачі оптимального керування з вибором конструктивних параметрів. Як окремий випадок, аналізується задача багатокритеріальної оптимізації лінійної системи за інтегральним квадратичним критерієм за умови заданого випадкового розподілу початкових збурень. Розв’язок ґрунтується на методі одночасного зведення двох виразно позитивних квадратичних форм до діагонального вигляду. Отримано аналітичні вирази, що дають змогу обчислити математичне очікування критерію для випадку багатовимірного нормального розподілу вектора початкових випадкових збурень. Сформульовано обернену задачу теорії стійкості: необхідно віднайти вектор структурних параметрів, що забезпечує стійкість системи та задане середнє значення інтегрального квадратичного критерію якості на множині початкових збурень. Розв’язання пропонується виконувати у два етапи. Спершу виводиться загальний розв’язок матричного рівняння Ляпунова в термінах елементів матриці системи. Для цього простір станів відображається на власний підпростір додатноозначеної матриці, що відповідає інтегральному квадратичному критерію. Встановлено, що загальний розв’язок зумовлений довільною кососиметричною матрицею або відповідним набором довільних сталих. З іншого боку, коли матриця системи лінійно залежить від вектора конструктивних параметрів, можливо сформулювати лінійну систему рівнянь відносно цих невідомих параметрів та довільних сталих, що фігурують у загальному розв’язку оберненої задачі стійкості. Загалом, така система є сумісною та допускає нескінченну кількість розв’язків, які задовольняють початкові вимоги до елементів симетричних матриць, присутніх у рівнянні Ляпунова.

Ключові слова: стійкість, інтегральний квадратичний функціонал, матричне рівняння Ляпунова, обернена задача стійкості, багатоцільові динамічні системи, лінійні стаціонарні системи, параметрична оптимізація.

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